A ferromagnetic chain in a random weak field

1996 J. Phys.: Condens. Matter 8 8379

(http://iopscience.iop.org/0953-8984/8/43/028)

J. Phys.: Condens. Matter **8** (1996) 8379–8383. Printed in the UK

A ferromagnetic chain in a random weak field

I Avgin†

Department of Physics, Erciyes University, 38039 Kayseri, Turkey

Received 16 May 1996, in final form 29 July 1996

Abstract. The harmonic magnon modes in a Heisenberg ferromagnetic chain in a random weak field are studied. The Lyapunov exponent for the uniform ($\boldsymbol{k} = \boldsymbol{0}$) mode is computed using the coherent potential approximation (CPA) in the weak-disorder limit. The CPA results are compared with the numerical and weak-disorder expansions of various random systems. We have found that the inverse localization length and the integrated density of states have anomalous power law behaviour as reported earlier. The CPA also reproduces the dispersion law for the same system, calculated by Pimentel and Stinchcombe using the real space renormalization scaling technique. A brief comment is also made for the uniform weak-field case.

1. Introduction

We have recently worked on the dynamics of one-dimensional spin glasses for binary and continuous distributions of the exchange interactions in the zero- and high-field limits using the coherent exchange approximation (CEA) [1] (a version of the coherent potential approximation (CPA); see the appendix). We have found that the CEA reproduces successfully the numerical and analytical results, particularly the anomalous power law behaviour of the dynamical quantities in the low-frequency regime. In this paper, a ferromagnetic Heisenberg chain in a random weak field is investigated using the CPA. The random field model is simpler than the spin glass model but they share complicated features such as frustration. This magnetic system is capable of capturing the fundamental physics behind a rich class of experimentally realizable random systems such as binary fluid mixtures in disordered porous media and some disordered Ising antiferromagnetics in an external field [2]. The behaviour of the random field system is determined by the battle between ferromagnetic exchange interaction J and local field h_i. In low dimensions, the ferromagnetic order is unstable against the formation of overturned large-spin droplets [2], but, for a weak field, a large reversed domain (opposing ferromagnetic order) appears with a small probability [3]. Thus the ground state has an effective ferromagnetic order over length scales longer than the harmonic magnon wavelength [3], that is $k^{-1} < 2J/h$ (for weak field $J/h \ll 1$, this criterion clearly holds [3]).

2. The CPA results for random field

We start with the linearized equation of motion for spin operator S^+ (equation (5) of [3])

$$(2 + \xi_n h - w)S_n^+ = S_{n-1}^+ + S_{n+1}^+ \tag{1}$$

† Present address: Kuleli Askeri Lisesi, Cengelkoy, Istanbul, Turkey.

where w and h are the magnon energy and strength of the random field (in units of J) respectively, and x_n is a random quantity taking (± 1) with equal probability. The $\xi_n h$ part in equation (1) can be associated with the potential fluctuations of the disordered alloy systems; hence, the CPA can be used. The CPA reduces the random medium to an effective medium characterized by a coherent field $h_c(w)$ which will be determined self-consistently by setting the configurationally averaged scattering matrix to zero [4] and the resulting self-consistent equation takes the form (see the appendix)

$$\frac{h - h_c(w)}{1 - (h - h_c(w))\overline{G}(w - h_c)} - \frac{h + h_c(w)}{1 + (h + h_c(w))\overline{G}(w - h_c)} = 0 \qquad (2a)$$

$$-h_c(w) + (h^2 - h_c^2(w))\overline{G}(w - h_c) = 0 \qquad (2b)$$

where

$$\overline{G}(w, h_c) = [(w - h_c)(w - h_c - 4)]^{-1/2} \qquad (3)$$

is the configurationally averaged Green function in one dimension [5] (however equations (2*a*) and (2*b*) are valid for any dimension). For vanishingly small w $h_c(w)$ is not a function of w. For relatively weak field and small w equation (2*b*) reduces to the well known limit of weak scattering [4]

$$h_c \cong h^2\overline{G}(-h_c) = h^2(4h_c)^{-1/2}. \qquad (4)$$

This can easily be solved and the complex solution (physically meaningful [5] since the real and imaginary parts of the solution are proportional to the Lyapunov exponent and the integrated density of states, respectively) is

$$h_c \cong (\mathrm{e}^{2\pi\mathrm{i}}h^2/2)^{2/3}. \qquad (5)$$

As discussed in [4], the equation (2) is obtained after the arithmetic average is performed; however it is not the arithmetic average that is representative of the ensemble in one dimension but rather the geometric average. Moreover, the geometric averaged coherent field h_c^G is related to h_c of equation (2) in that

$$h_c^G \approx \tfrac{1}{2}h_c \qquad (6)$$

for the weak-field limit [4].

The dynamical properties of a disordered chain can be obtained fruitfully from the studies of the complex Lyapunov exponent γ [6]. Thouless [7] established that $\mathrm{Re}\,\gamma$ is the inverse localization length and $-(\mathrm{Im}\,\gamma)/\pi$ is the integrated density of states. For our context, the Lyapunov exponent is given by [7, 1]

$$\gamma(w - h_c^G) = \int \overline{G}(w - h_c^G)\,\mathrm{d}w. \qquad (7)$$

Using equations (5) and (7), the complex Lyapunov exponents for the uniform ($\boldsymbol{k} = \mathbf{0}$) harmonic magnon mode can be obtained

$$\gamma(h) = -(h_c^G)^{1/2} = 2^{-5/6}\exp(5\pi\mathrm{i}/3)h^{2/3} \qquad (8)$$

whose real and imaginary parts are given by

$$\mathrm{Re}\,\gamma = 2^{-5/6}\cos(5\pi/3)h^{2/3} \qquad (9)$$

$$-\frac{\mathrm{Im}\,\gamma}{\pi} = \pi^{-1}2^{-5/6}\sin(5\pi/3)h^{2/3}. \qquad (10)$$

To compare the CPA results with the disordered tight-binding chain and numerical simulation results, we should, first, use the analogy given in [8] and then identify the random potential. Equation (1) is analogous to the tight-binding chain [8, 1]

$$(E - \lambda V_n)\psi_n = \psi_{n+1} + \psi_{n-1} \tag{11}$$

if $E = 2$, $w = 0$ and random potential $\lambda V_n = -\xi_n h$. The integrated density of states and inverse localization length are proportional to $\langle \lambda^2 V^2 \rangle^{1/3}$ for even distribution of random potential [6]. To compare the results, we set $\mathrm{Re}\,\gamma = Ah^x$ and $-(\mathrm{Im}\,\gamma)/\pi = Bh^x$; A, B and x are given in table 1 where one can see a good agreement between the CPA results and the numerical results [8] and those of Derrida and Gardner [6].

Table 1. The integrated density of states and the inverse localization length obtained by various methods.

	Numerical results [1, 8]	Derrida and Gardner [6]	CPA	CEA [1]
A	0.162	0.159	0.155	0.164
B	0.285	0.289	0.281	0.297
x	2/3	2/3	2/3	2/3

For small magnon damping, as is the case for small $\boldsymbol{k}$ and h, we can obtain the magnon dispersion relation from the poles of the $\boldsymbol{k}$ dependent Green functions:

$$w_k = h_c^G + k^2 \sim \{(h/k^{3/2})^{4/3} + 1\}k^2 \tag{12}$$

and

$$w_k \sim \begin{cases} k^2 & \text{for } hk^{-3/2} \ll 1 \\ h^{4/3} & \text{for } hk^{-3/2} \gg 1. \end{cases} \tag{13}$$

This dispersion relation was first obtained by Pimentel and Stinchcombe [3] using the real space renormalization technique. Equation (13) gives the usual ferromagnetic dispersion law for very small h, and for the other limit the dynamics are governed by a nontrivial power law of the field. Moreover, the cross-over can be observed [3] since $k^{-1} \sim h^{-2/3} < h^{-1}$, the criteria for the existence of magnons, are fulfilled.

3. A spin glass in uniform field

Here we consider the case where h is weak but nonrandom and the exchange interactions are random and equally distributed $(\pm J)$. This is a spin glass system and the equation of motion is given by [3]

$$[2 - \xi_n(w - h)]u_n = u_{n+1} + u_{n-1} \tag{14}$$

where $u_n = \xi_n S_n^+$ and $\langle S_n^Z \rangle = \xi_n$ (thus the ground state is random). For $w = 0$, the equations of motion for the random field and the spin glass are equivalent. Since the disorder is in the exchange interactions we can use the CEA. Equation (14) has the same form as the spin glass in the high-field case; the CEA results of [1] can be used. They are shown in table 1 (however, notice that the random potential for equation (14) is $\xi_n(w - h)$ and the coefficients A (or B) $(w - h)^x$). The dispersion law for long-wavelength excitations can be written as

$$w - h \sim J_c(w - h)k^2 \tag{15}$$

in which the coherent exchange [1] $J_c(w-h) \sim [-(w-h)]^{-1/3}$ for low $w-h$ (see the appendix). It takes the form

$$w_k \sim \begin{cases} k^{3/2} & \text{for } hk^{-3/2} \ll 1 \\ h & \text{for } hk^{-3/2} \gg 1. \end{cases} \tag{16}$$

For the small-h limit, the zero-field spin glass dispersion relation is recovered [3, 8, 1].

4. Summary and conclusion

In this paper we have discussed a ferromagnetic chain in a random weak field using the CPA. We found that the CPA results, particularly the anomalous power law behaviour, are in good agreement with the numerical and exact perturbation calculations in the weak-disorder limit. A brief comment on spin glasses in a small uniform field is also made. We are in the process of calculating the full spectrum for one and higher dimensions.

Acknowledgments

I have benefited greatly from discussions with Professor D L Huber and Dr U Yahsi. This work is sponsored by the Scientific and Technical Research Council of Turkey (Tubitak).

Appendix

The CPA was originally developed to account for diagonal disorder of the electronic problem [9]. The method was also generalized to treat random magnetic systems [9]. Let us consider the Heisenberg Hamiltonian in a magnetic field

$$H = -\sum_n J_{n,n+1} \boldsymbol{S}_n \boldsymbol{S}_{n+1} - \sum_n \boldsymbol{h}_n \boldsymbol{S}_n \tag{A1}$$

where $\boldsymbol{S}$, $J_{n,n+1}$ and $\boldsymbol{h}_n$ are the spin, exchange integral and site dependent magnetic field, respectively. First consider that the exchange integral is uniform but the field is site dependent and random; then the second part of equation (A1) acts like a diagonal disorder of the electronic problem. We then can employ the CPA whose configurationally averaged scattering matrix is of the form

$$\langle T \rangle = \int \mathrm{d}h \; p(h) \frac{(h - h_c(w))}{1 - (h - h_c(w))\overline{G}(w - h_c)} \tag{A2}$$

where $p(h)$ is the distributions of the random field. Second consider that the field is uniform but the exchange integral is random. This is an off-diagonal (bond) disorder and, to account for this, a modified version of the CPA to the bond problem where the coherent potential is replaced by a coherent exchange was developed [10]. For this case, the configurationally averaged scattering matrix is given by

$$\langle T \rangle = \int \mathrm{d}J \; p(J) \frac{(J - J_c(w-h))}{1 - (J - J_c(w-h))F(w-h, J_c)} \tag{A3}$$

where $p(J)$ is the distributions of the exchange integrals and $F(w-h, J_c) = -1/J_c + [(w-h)/J_c]\overline{G}$. In this paper only symmetric distributions of the field and the exchange integral are considered and $p(x) = \frac{1}{2}[\delta(x - x_c) + \delta(x - x_c)]$ where x represents h or J.

References

[1] Avgin I and Huber D L 1993 *Phys. Rev.* B **48** 13 625
Boukahil A and Huber D L 1994 *Phys. Rev.* B **50** 2978
Avgin I, Boukahil A and Huber D L 1994 *Trends Stat. Phys.* **1** 97

[2] Daniel S F, Geoffrey M G and Anil K 1988 *Phys. Today* **Dec** 59

[3] Pimentel I R and Stinchcombe R B 1989 *Phys. Rev.* B **40** 4947

[4] Economou E N, Soukoulis C M and Zdetsis A D 1984 *Phys. Rev.* B **30** 1686
Economou E N 1983 *Green's Functions in Quantum Physics* (Heidlberg: Springer) p 142

[5] Boukahil A 1991 *PhD Thesis* University of Wisconsin Madison

[6] Derrida B and Gardner E 1984 *J. Physique* **45** 1283

[7] Thouless D J 1972 *J. Phys. C: Solid State Phys.* **5** 77

[8] Boukahil A and Huber D L 1989 *Phys. Rev.* B **40** 4638

[9] Elliot R J, Krumhansl J A and Leath P L 1974 *Rev. Mod. Phys.* **46** 465 and references therein

[10] Tahir-Kheli R A 1972 *Phys. Rev.* B **6** 2808